Bunch Response Functions

Alexey Burov

Jan 20 2004

SB Amplitudes

- For the air-bag distribution $z = \hat{z}\cos\varphi$ the offset can be expressed in terms of the SB amplitudes

$$x(\varphi,t) = \sum_l A_l \exp(il\varphi + i\chi\cos\varphi - i\Omega_l t) + c.c.; \quad \chi \equiv \frac{\xi\hat{z}}{R\eta}; \quad \Omega_l = \omega_b + l\omega_s$$

- Inverse transformation follows as

$$A_l = \oint \frac{d\varphi}{4\pi}\left[x(\varphi,t) + i\dot{x}(\varphi,t)/\omega_b\right]\exp(-il\varphi - i\chi\cos\varphi + i\Omega_l t);$$

- A single kick distributed along the bunch as

$$\delta\dot{x} = u\cos(k_k z + \theta_k) = u\cos(q_k\cos\varphi + \theta_k)$$

gives rise to the amplitude perturbation

$$\delta A_l = \frac{iu}{4\omega_b} i^{-l}\langle k|l\rangle; \quad \langle k|l\rangle = e^{i\theta_k}J_l(\chi - q_k) + e^{-i\theta_k}J_l(\chi + q_k)$$

where the bracket factor $\langle k\,|\,l\rangle$ describes the kicker visibility for the mode l.

Offset distribution

- In case when there is an infinite number of kicks applied with a given frequency as $u_n = V\cos(\omega t_n + \psi)\,;\quad t_n = nT_0$, they are summarized giving

$$\delta A_l = \frac{iV}{4\omega_b} i^{-l} \langle k | l \rangle \frac{1}{2} \frac{\exp(i(\Omega_l - \omega)t_n - i\psi)}{1 - \exp(-i(\Omega_l - \omega)T_0)}$$

- Taking into account that only small interval of frequencies around the eigen-frequency Ω_l is of interest, the exponent in the denominator is expanded leading to

$$x(\varphi, t) = \mathrm{Re}\left\{ \frac{V e^{-i(\omega t + \psi)}}{4\omega_b} i^{-l} \sum_l \langle k | l \rangle \frac{\exp(il\varphi + i\chi\cos\varphi)}{\Omega_l - \omega} \right\}$$

where all the eigen-frequencies lie in the lower half-plane.

Measured Response

- Let the pickup measurement be modeled as

$$R = \oint \frac{d\varphi}{4\pi}\left(x(\varphi) + x(-\varphi)\right)\cos(q_p \cos\varphi + \theta_p)$$

- Integration results in

$$R(t) = \frac{V\sqrt{\beta_k \beta_p}}{8C}\,\mathrm{Re}\left[e^{-i(\omega t+\psi)} G(\omega)\right]; \quad G(\omega) = \sum_l \frac{\langle k \mid l\rangle\langle l \mid p\rangle}{\Omega_l - \omega},$$

with the bracket

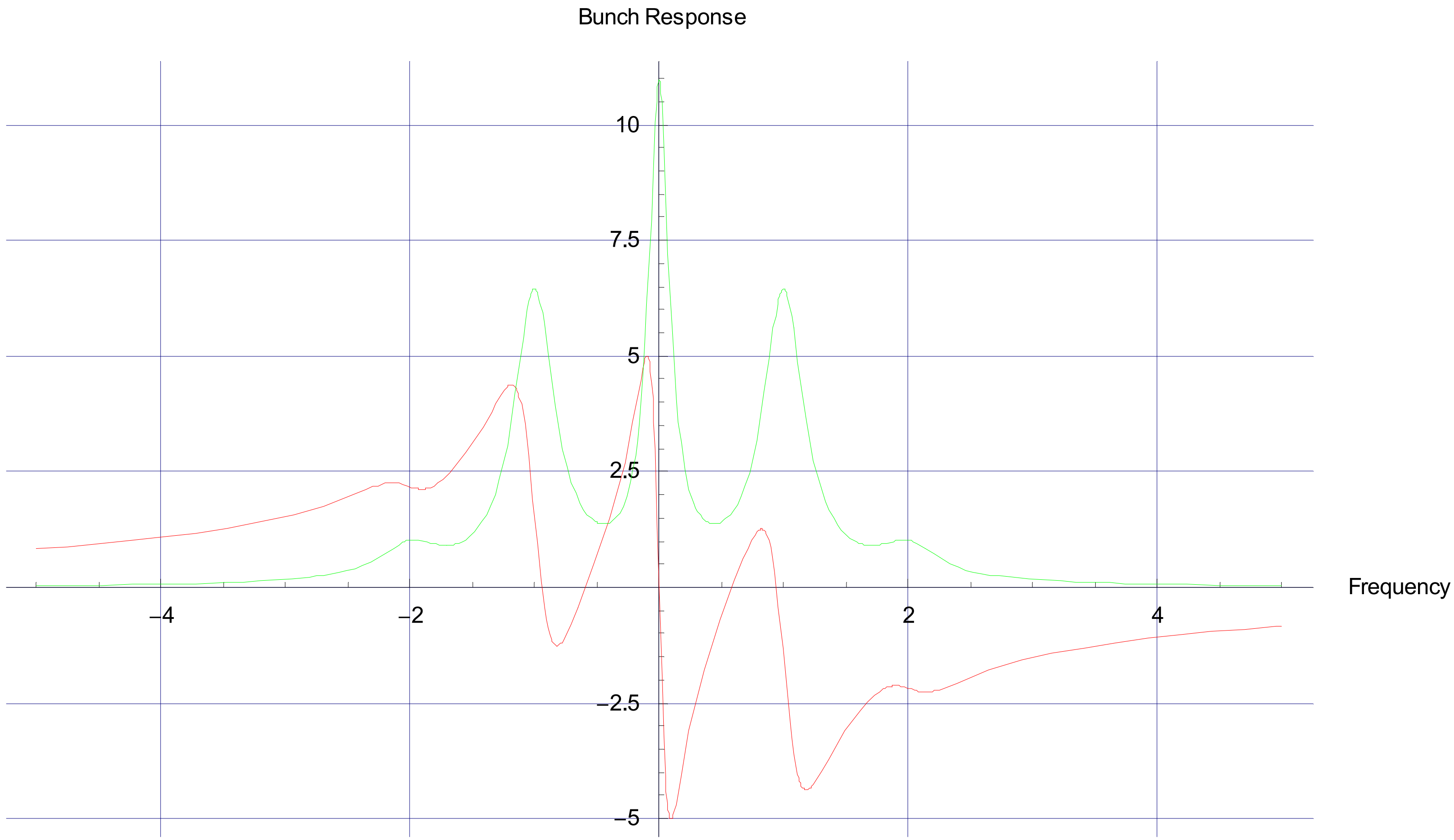
$$\langle l \mid p\rangle = e^{-i\theta_p} J_l(\chi - q_p) + e^{i\theta_p} J_l(\chi + q_p)$$

describing how effective is the mode seen by the pickup.

Air-bag Response

- Air-bag response functions (Re and Im parts) for

$N/N_{th} \ll 1$ and $\mathbf{Im}\,\Omega_l = (0.1\,|\,l\,|+0.1)\omega_s$



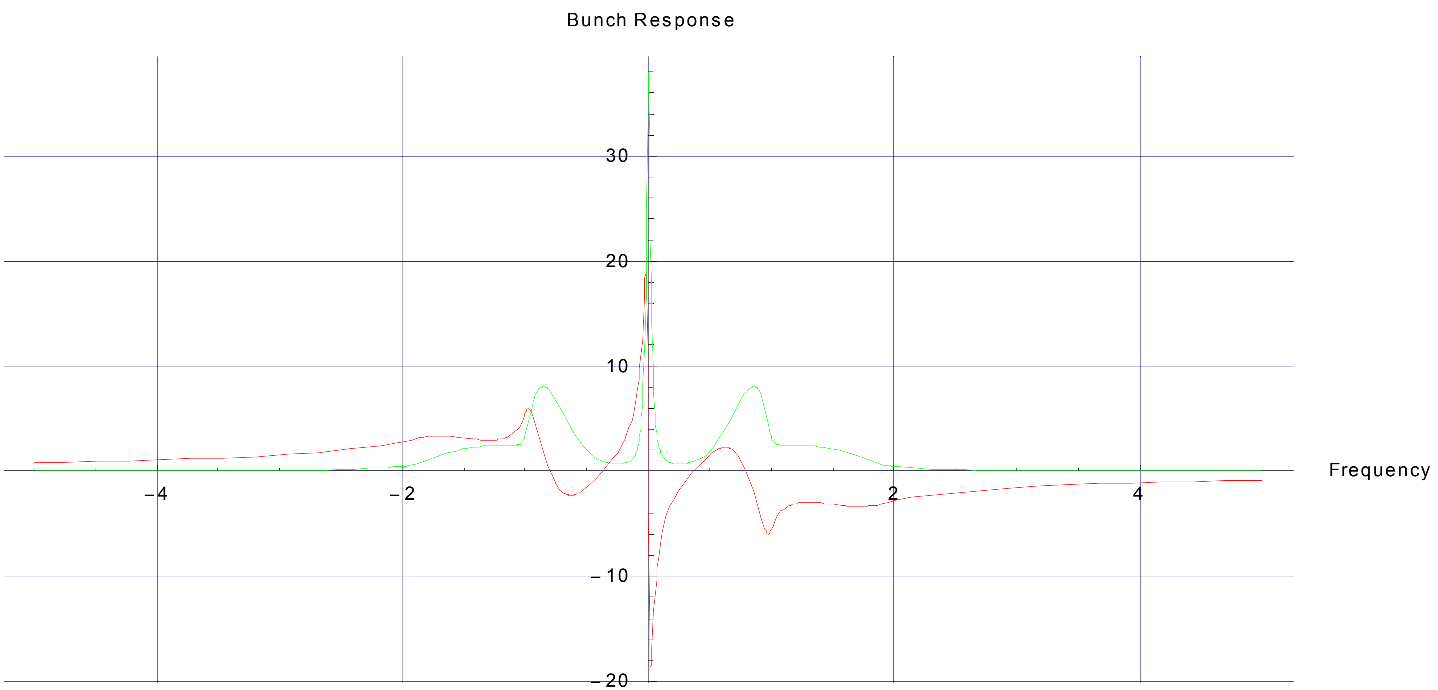
Parabolic distribution with non-linearity

- A model distribution

$$f(\hat{z}),\ \omega_s(\hat{z}) \propto 1-\hat{z}^2/z_m^2$$

response is calculated by integration of it's air-bags with

$$\Omega_l = l\omega_s(\hat{z}) - 0.02i\,\omega_s(0)$$



If not far from the threshold

- When the bunch population is not far from the strong head-tail threshold (TMCI), the impedance modifies the response:
  - They are not symmetric any more
  - The peaks are shifted
  - The peaks are not equidistant
- Although the response can be formally expressed in the same way,

$$R(t) = \frac{V\sqrt{\beta_k \beta_p}}{8C} \mathrm{Re}\left[e^{-i(\omega t+\psi)} G(\omega)\right]; \quad G(\omega) = \sum_l \frac{\langle k | l \rangle \langle l | p \rangle}{\Omega_l - \omega},$$

the visibility form-factors are not as easy to calculate.